



## **STANDARDS ALIGNMENT ANALYSIS FOR KENTUCKY DRAFT (JUNE 15, 2005)**

### **Introduction**

At the request of the Kentucky Department of Education, Achieve, Inc. conducted an analysis of several of the state's mathematics standards documents, comparing them with Achieve's American Diploma Project (ADP) benchmarks to determine the degree of alignment between them. The state of Kentucky has embarked upon a concerted effort to refine and strengthen its mathematics expectations for middle school and high school students and, at the same time, align those expectations with what national research shows students need to know and be able to do if they are to be prepared for college and the world of work. An impetus for Kentucky's work has been its involvement in the American Diploma Project (ADP), a collaboration of Achieve, Inc., The Education Trust, and the Thomas B. Fordham Foundation designed to establish a strong link between the secondary and postsecondary communities. These ADP partners worked closely with K-12, postsecondary, and business leaders in five states (Indiana, Kentucky, Massachusetts, Nevada, and Texas) to identify the mathematics and English language arts knowledge and skills needed for success in both college and work.

For purposes of this analysis, Achieve was asked to analyze the mathematics expectations contained within the following documents and determine the degree to which these documents align with and support one another.

- Program of Studies for High School (last updated June 22, 2004)
- Program of Studies for Middle Level Mathematics (last updated June 22, 2004)
- High School Mathematics Core Content for Assessment (May 12, 2005 draft)
- Middle School Mathematics Content for Assessment (May 12, 2005 draft)
- Kentucky Statewide College-Readiness Standards in Mathematics (Attachment 3 to the Council on Postsecondary Education's Statewide Public Postsecondary Placement Policy dated November 8, 2004)
- Achieve's American Diploma Project (ADP) Mathematics Benchmarks (2004)

This report communicates the findings of this comparative analysis. A brief general summary provides the broad findings and recommendations, followed by more detailed information. In-depth analyses include a comparison of KY's College Readiness Standards in Mathematics with the ADP mathematics benchmarks and an alignment analysis of the KY Mathematics Program of Studies and the draft Core Content for Assessment with KY's Statewide College Readiness Standards in Mathematics and the ADP Benchmarks. A detailed "side-by-side" chart that aligns comparable expectations from each of these documents is also included as part of this study.

## **General Summary of Findings and Recommendations**

- There is strong alignment between the Kentucky Statewide College Readiness Standards in Mathematics and Achieve's American Diploma Project (ADP) benchmarks in mathematics when the two documents are compared in their entirety. Differences between the two documents arise when an examination is made of the expectations each defines as essential for all students. The Kentucky document defines three levels of expectation, with the first level being noted as essential gateway mathematical skills that students should have to avoid placement into remedial courses and succeed in an entry-level college course. The ADP mathematics benchmarks have two levels of expectation with all benchmarks except those noted with asterisks being deemed essential for all students. The major differences between the two documents are as follow:
  - The ADP benchmarks define expectations with respect to technology for all students, while the KY College Readiness Standards define these standards as valuable but able to be acquired in college.
  - The ADP benchmarks define as essential for all students a multitude of expectations with respect to data analysis, probability, and statistics. KY's College Readiness Standards classify those expectations that extend beyond basic data display, data interpretation, and summary statistics as knowledge that can be acquired in college.
  - The ADP benchmarks place a greater emphasis on proof and construction than the KY College Readiness Standards do. The ADP benchmarks call for all students to be able to use geometric properties to prove and to perform constructions, in addition to being able to apply such properties to solve problems. The KY College Readiness Standards emphasize identification and application, in lieu of proof and construction, with any expectations addressing proof identified as essential only for those students whose intended majors require calculus and who expect to begin college taking calculus. The KY College Readiness Standards do not reference geometric constructions.

The ADP benchmarks and the KY College Readiness Standards have somewhat different purposes that help explain the aforementioned variations. While the KY document is focused on college readiness, the ADP document has the broader mission of college and workplace readiness. While a laudatory goal is to prepare all students so they have the opportunity to attend college, it is important that high schools also provide students with all of the skills they need to be prepared for a high performance workplace should they choose or be unable to attend college upon graduation. It is important that all students have facility with applying calculator and computer technology to solve problems. It is also important that all students have a solid foundation in data analysis and statistics so that they can make sound judgments in their personal and professional lives and be informed and critical consumers of mass media. Likewise, although students pursuing a

major that is not mathematics-intensive and those entering directly into the workplace may not need to be able to do formal geometric proofs and constructions, it is important that they come prepared with the strong sense of mathematical reasoning that such activities impart. It is important that Kentucky's K-12 standards—as they are revised and refined—embed these aspects of content and performance that are missing for all students in KY's College Readiness Standards.

- The alignment between KY's K-12 documents (Program of Studies and Core Content for Assessment), the KY Statewide College Readiness Standards, and the ADP benchmarks is less clear. Of the two K-12 documents, the Core Content for Assessment is more comprehensive and more appropriately rigorous—and better aligned with both the KY College Readiness Standards and the ADP benchmarks. KY's Program of Studies and its Core Content for Assessment were drafted at different points in time and for different purposes. The intent of the Program of Studies—at the high school level—is to define the minimum content for the courses that comprise two of the three mathematics credits required for high school graduation. The purpose of the Core Content for Assessment is to define content that is essential for all students to know and that is eligible for inclusion on the state assessment—administered in high school at grade 11. One would hope that the content statements from these two documents would align reasonably well, but unfortunately this is not always the case. The result is a set of documents that offer varying perspectives on what is required of Kentucky high school graduates, hence creating the potential for confusion on the part of educators and the community. Specific details on the alignment of these various documents are provided in the body of this report, organized by content domain (i.e., Number Properties and Operations, Measurement, Geometry, Data Analysis and Probability, and Algebraic Thinking).
- To improve the coherence of the Program of Studies, Core Content for Assessment, and College Readiness Standards—and to ensure alignment of all of these with the ADP mathematics benchmarks—Achieve offers the following general suggestions and recommendations:
  - For the K-12 documents, consideration should be given to creating one document that clearly communicates the level of mathematics knowledge and skills expected of Kentucky students when they graduate. This document could be designed in such a way as to include student standards, those aspects of the student standards deemed to be eligible for inclusion on statewide assessments, and even parameters/limits for assessment items. Maryland and Pennsylvania have both done work in the area, and their documents might serve as models for Kentucky. The existence of one document would go a long way in achieving a consistent and clear voice about student expectations and make alignment with postsecondary expectations a more realistic task.

- Achieve is currently working to “backmap” its ADP benchmarks in mathematics—creating a progression of content expectations by content domain that will extend over the high school years. Once these content progressions are completed, Achieve plans to parse these expectations into course sequences—both a traditional course sequence and an integrated sequence—to model how high school mathematics courses could look that culminate in meeting ADP’s benchmarks. At the same time, Achieve is working to ensure that this “backmapped” sequence aligns with the expectations Achieve set out earlier in its Mathematics Achievement Partnerships’s *Foundations for Success* document, which defines mathematics expectations for the middle grades. Kentucky might find Achieve’s work helpful in its revision process and/or to validate the work that it does independently.
- As work continues to define a coherent set of student expectations, consideration should be given to creating examples to help clarify the intent and the level of rigor of the expectations. The ADP benchmarks and a number of state standards documents include examples as part of their standards. Pennsylvania—referenced earlier as a model for combining student expectations with assessment parameters—includes student expectations, assessment parameters, and examples in one document. An example of one page of this document is provided on the following page.

# Sample from Pennsylvania's Math Grade 11 Assessment Anchors and Eligible Content

## **ASSESSMENT ANCHOR**

**M11.A.1 Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**

## **ELIGIBLE CONTENT**

**M11.A.1.1** Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).  
*Reference: 2.1.8.A, 2.1.8.B, 2.1.11.A*

**M11.A.1.1.1** Represent and/or use fractions as decimals and percents (item may ask for 2 of any of these 3 – change percent to fraction, change fraction to decimal, etc).  
**M11.A.1.1.2** Find the square root of an integer using either a calculator or estimation (integer may or may not be a perfect square – answer may be a range of values).  
**M11.A.1.1.3** Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).  
**M11.A.1.1.4** Simplify square roots (e.g., the square root of 24).

## **EXAMPLE ITEMS**

- The diameter of a red blood cell, in inches, is  $3 \times 10^{-4}$ . This expression is the same as which of the following numbers?

- A. 0.00003
- \* B. 0.0003
- C. 0.003
- D. 3,000
- E. 30,000

(NAEP)

- $\frac{6 \times 10^3}{3 \times 10^5} =$

- A.  $0.5 \times 10^2$
- B.  $2 \times 10^2$
- C.  $2 \times 10^{0.6}$
- D.  $0.5 \times 10^{-2}$
- \* E.  $2 \times 10^{-2}$

(NAEP)

- As revisions and refinements are made, Kentucky should reconsider the wording of some of its expectations to make sure they are clear, concise, and pegged at the desirable degree of rigor. Content statements should be succinct enough that their intent and level of rigor can be described easily by examples. A standard such as one now in the Program of Studies indicates that students should “use the skills learned to solve linear equations and inequalities to solve numerically, graphically, or symbolically non-linear equations such as quadratic and exponential equations.” This is a very broad standard, encompassing a multitude of content and performance expectations, that could be broken into more manageable expectations whose intent and level of rigor could be exemplified by sample problems. This would make for a more understandable and usable document. Care also needs to be taken in clearly articulating parameters and limitations on the expectations, for both the purposes of instruction and assessment.

### **Alignment of the KY Statewide College-Readiness Standards in Mathematics with the ADP Mathematics Benchmarks**

The KY Statewide College-Readiness Standards in Mathematics align closely with Achieve’s ADP mathematics benchmarks when the documents are analyzed in their entirety. Since the KY standards identify three levels of expectation and the ADP benchmarks define two levels of expectation, there are variations between the two documents in the placement of specific content. At least some of this variation is attributable to the fact that the ADP benchmarks are intended to define college and workplace ready skills, while KY’s standards are targeted at college readiness. The differences between these two documents are detailed below.

- A number of ADP benchmarks that are identified for all students are placed in the KY College-Readiness Standards at Level 2, meaning that KY defines them as valuable skills that can be acquired in a college-level credit-bearing course. The most important of these are those ADP benchmarks devoted to the use of technology, including calculators and computers, plus a significant proportion of the benchmarks related to data analysis, statistics and probability. In particular, the KY College Readiness Standards tend to define the expectations that extend beyond basic data display and interpretation and summary statistics as Level 2 expectations. An itemization of the ADP benchmarks for all students that are classified as Level 2 in the KY College Readiness Standards are as follows:
  - I4 (understanding the capabilities and limitations of technology in solving problems)
  - I4.1 (using calculators appropriately and accurately to solve problems)
  - I4.2 (using graphing calculators and computer spreadsheets)
  - J5.6 (recognizing and solving problems that can be modeled using a finite geometric series)

- K9 (visualizing solids and surfaces in 3-D space when given 2-D representations)
  - L1.4 (comparing data sets using graphs and summary statistics)
  - L1.5 (creating scatter plots and using them to analyze relationships)
  - L1.6 (knowing characteristics of the normal curve)
  - L2 (explaining and critiquing ways of presenting/using information)
  - L2.1 (evaluating media reports based on data)
  - L2.2 (explaining misleading uses of data)
  - L2.3 (understanding the difference between correlation and causation)
  - L3 (using data to draw inferences, make predictions, justify conclusions)
  - L3.1 (explaining the impact of sampling, bias, and phrasing of questions)
  - L3.2 (designing simple experiments)
  - L3.3 (explaining the differences between randomized experiments and observational studies)
  - L3.4 (determining and understanding the line of best fit, using technology)
  - L4 (explaining and applying probability concepts)
  - L4.1 (explaining probability as quantifying likelihood)
  - L4.2 (explaining relative frequency as an estimate of probability)
  - L4.3 (explaining application of the law of large numbers)
  - L4.4 (applying probability concepts to calculate simple probabilities)
  - L4.5 (applying probability concepts to practical situations)
- Only one ADP benchmark identified for all students is defined as a Level 3 expectation in KY's College Readiness Standards, meaning that KY perceives this as essential only for students who intend majors that require calculus. This benchmark (K6) addresses the use of rigid motions, or geometric transformations, to determine whether two figures are congruent and to create and analyze geometric designs.
  - The asterisked ADP benchmarks—representing content that is recommended for all students but required for those intending to take calculus in college—tend to be partitioned into KY's Level 2 and Level 3 College-Readiness Standards. Only two benchmarks are asterisked in the ADP document but defined in the KY document to be essential gateway skills (Level 1) for KY students.
    - Asterisked ADP Benchmarks Identified as Level 1 in KY's College-Readiness Standards
      - ✓ J1.2 (understanding and applying the properties of rational exponents)
      - ✓ Part of K10.4 (finding the equation of a circle, given its center and radius).

- Asterisked ADP Benchmarks Identified as Level 2 in KY's College-Readiness Standards
  - ✓ J4.6 (understanding and graphing ellipses and hyperbolas whose axes are parallel to the  $x$  and  $y$  axes)
  - ✓ J.5.5 (modeling with exponential functions in instances requiring facility with logarithms, e.g., exponential growth and decay problems)
  - ✓ J1.7 (deriving and using formulas for finite arithmetic and geometric series and finding the sum of an infinite geometric series with a common ratio between -1 and 1)
  
- Asterisked ADP Benchmarks Identified as Level 3 in KY's College-Readiness Standards
  - ✓ J2.2 (determining domain of a function)
  - ✓ J2.4 (combining functions)
  - ✓ J2.5 (identifying and understanding inverses of functions)
  - ✓ J2.6 (knowing the inverse relationship between exponential functions and logarithms and proving/using basic related properties)
  - ✓ J6 (understanding the binomial theorem and its connections to combinatorics, Pascal's triangle and probability)
  - ✓ Part of K10.4 (finding the center and radius of a circle, given its equation)
  - ✓ K12 and K12.1 (understanding and using periodic functions)
  - ✓ K12.2 (knowing and using basic trigonometric identities)
  - ✓ K12.3 (graphing trigonometric functions and their reciprocals)
  - ✓ K12.4 (knowing and using the law of sines and law of cosines)
  
- In a number of instances, KY has truncated or otherwise modified the expectations defined in some of the ADP benchmarks. In particular, a number of changes were made in ADP geometry benchmarks that called for all students to be able to prove and do constructions. Such expectations were generally altered to state that all college-ready KY students need to be able to identify and apply properties, but not to use them in proofs or constructions.
  - J4.4 (revised to eliminate the expectation that students graph the solution set of a system of linear inequalities; the KY expectation is limited to graphing the solution set of a linear inequality.)
  - K1.2 (revised to state that all students should be able to state and use key basic geometric theorems, with proof limited to the Level 3 expectations)
  - K2 (revised to include properties—in addition to definitions—but to eliminate the expectation of proofs or constructions related to lines and angles)

- K2.1 (revised to eliminate the expectation of proofs or constructions related to parallel lines)
  - K2.2 (revised to eliminate the expectation of proofs or constructions related to perpendicular lines)
  - K2.3 (revised to eliminate the expectation of proofs or constructions related to angles)
  - K3 (revised to state that all students should understand—not just know—the basic theorems about congruent and similar triangles, with proof limited to the Level 3 expectations)
  - K7 (revised so that similarity is addressed, but not scale factors specifically)
  - K4 (revised to state that all students should understand—not just know—the definitions and basic properties of circles, with proof limited to the Level 3 expectations)
- Only three of the ADP benchmarks are not included in the KY College-Readiness Standards. The first of these three is an asterisked expectation in ADP.
    - J3.4 (solving systems of three linear equations in three variables)
    - K1.3 (recognizing non-Euclidean geometry)
    - K11.3 (understanding and using the formula to find the area of a triangle given the lengths of two sides and the included angle)
  - The ADP benchmarks define nine mathematical reasoning skills that students graduating from high school need—and that are woven throughout the ADP benchmarks. The KY College Readiness Standards do not provide a separate listing of mathematical reasoning skills.

**Alignment of the KY Mathematics Program of Studies and Draft Core Content for Assessment with KY's Statewide College Readiness Standards in Mathematics and the ADP Benchmarks**

In addition to analyzing KY's College Readiness Standards in Mathematics with respect to the ADP benchmarks, Achieve also analyzed two sets of KY documents intended to define mathematics expectations for middle school and high school students in the state.

- The Program of Studies for middle level and high school mathematics is defined by grade for grades six through eight and by course for Algebra I and geometry. At the high school level, the Program of Studies defines the minimum content for the courses that comprise two of the three mathematics credits required for high school graduation. The third course—a mathematics elective—is not defined in the Program of Studies.
- The draft Mathematics Core Content for Assessment for middle school and high school (May 12, 2005 draft) represents content the state has identified as essential for all students and that will be included on the state assessment. It is intended to be used with, not instead of, the Program of Studies to focus development of test

items for the Kentucky Core Content Test. Bolded content statements within the documents are eligible for inclusion on the state assessment, while italicized content statements are designated as supporting content not to be included on the state test.

The goal is to have a comprehensive and coherent system of mathematics expectations through grade 12 that is also in alignment with what students need to be prepared for college and the world of work. This analysis by Achieve is intended to assist Kentucky by examining the extent to which its current standards documents are in alignment and by identifying ways in which the documents might be brought into closer alignment. In order to clarify the similarities and differences of these four documents (Program of Studies, Core Content for Assessment, College Readiness Standards, and the ADP benchmarks), the findings of the analysis will be organized by content domain as structured in the Core Content for Assessment document (i.e., Number Properties and Operations, Measurement, Geometry, Data Analysis and Probability, and Algebraic Thinking).

## NUMBER PROPERTIES AND OPERATIONS

- The ADP benchmarks include expectations with respect to technology—including graphing calculators and computer spreadsheets—but neither the Program of Studies nor the Core Content for Assessment explicitly addresses technology within their content statements. The Program of Studies does mention calculators in its introductory material, and the Core Content for Assessment mentions students using calculators as one tool for checking their work. The College Readiness Standards address technology only at Level 2, indicating that such skills can be acquired while in college. Achieve recommends that KY include expectations with respect to technology in its middle school/high school expectations since all high school graduates—whether entering college or the world of work—need skills in using calculator and computer technology to solve problems.
- A number of the expectations that both the ADP and KY College Readiness Standards set with respect to Number Properties and Operations are defined in KY’s standards for the middle grades. The Program of Studies for the middle grades mentions fractions, decimals, percents, integers, rational numbers, irrational numbers, proportions, rates, properties, and number theory concepts. It is not clear, however—given the vague wording of some of these expectations—whether students will indeed be prepared for what is expected of them as high school students. The middle-level expectations in the Program of Studies reference students being able to “use percents, decimals, integers, and fractions,” “relate irrational and rational numbers,” and “use irrational numbers.” Without examples and/or more specific language to clarify what is meant by “use” and “relate,” it is difficult to determine the level of rigor of these expectations. The expectations in the Core Content for Assessment more clearly articulate that students exiting middle school are expected to be able to solve real-world and mathematical problems involving rational numbers, use proportional reasoning to

solve problems, and to have a good enough sense of rational and irrational numbers that they can order and compare them.

- The expectations for high school tend to be more rigorous in the Core Content for Assessment than they are in the Program of Studies. This difference in the level of expectation makes it unclear what the expectations actually are. For example, the high school expectations related to Number Properties and Operations in the Program of Studies appear to be limited to using proportional reasoning to formulate and solve problems and using the order of operations. These are concepts that can easily be embedded into an Algebra I course—which is one of the two courses defined in the Program of Studies. The Core Content for Assessment makes it clear that the expectation in high school is that students expand their background in rational numbers to the real numbers. It is also clearly articulated in the Core Content for Assessment that high school students are to understand integer exponents, square and cube roots, factorials, various representations of large and small numbers, absolute value, and to be able to relate the concept of proportional reasoning to slope. These concepts are not spelled out in the Program of Studies, and it is also the case that the Program of Studies makes no reference to absolute value or scientific notation—which are concepts addressed in the other three documents examined.
- Since scientific notation is not mentioned in the Program of Studies and is only a supporting content statement in the Core Content for Assessment, it is not clear when students will be held accountable for knowing this content. This is the case with other supporting content statements in the Core Content for Assessment document (e.g., the expectation that students use order relations to represent problems using real numbers), so Kentucky would be well served to re-examine such expectations.
- Absolute value is addressed solidly in the ADP benchmarks and KY's College Readiness Standards but only briefly mentioned in the Core Content for Assessment. It is not mentioned at all in the Program of Studies.
- Both the ADP benchmarks and KY's College Readiness Standards set the expectation that students understand the rationale for extending the number system to include complex numbers and that they be able to define and give examples of complex numbers. Complex numbers are not referenced in the Program of Studies or the Core Content for Assessment, and if the expectation is that all college-ready students have this level of facility with complex numbers, it is important to include it in students' high school mathematics experiences.
- The only one of the documents analyzed to deal explicitly with estimation is the Core Content for Assessment. Estimation is assessable at grade 8 but not in grade 10.

## MEASUREMENT

- The Measurement expectations in the four documents tend to align well. All documents expect students to measure, to convert from one unit to another, and to find perimeter, area, volume, and surface area. Kentucky is advised to review the geometric shapes referenced in its multiple standards documents to ensure that a

clear message about expectations is sent to users of the document. For example, the ADP benchmarks, the College Readiness Standards, and the Core Content for Assessment all reference finding the surface area and volume of prisms, pyramids, cones and spheres. The Core Content for Assessment also references cylinders, and the Program of Studies mentions (in the middle-level standards) cubes, cylinders, and prisms. At the high school level, the expectation in the Program of Studies is worded much more generally, just referencing finding the surface area of solids.

- All documents except the Program of Studies expect students to understand how a change in a dimension of a geometric shape (e.g., effect of a scale factor) affects its perimeter, area, and volume.

## GEOMETRY

- The ADP benchmarks and the KY College Readiness Standards (at Level 1) expect students to understand the need for and give examples of geometric definitions, axioms, and theorems. This is not addressed in either the Program of Studies or the Core Content for Assessment.
- The ADP benchmarks differ from all three of the Kentucky documents in the emphasis they place on geometric proof and constructions. In particular, ADP specifies that students are to be able to prove such basic theorems as the Pythagorean theorem, the sum of the angles of a triangle is 180 degrees, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length. The ADP benchmarks also set the expectation that students be able to prove theorems and perform constructions related to lines and angles, parallel lines, perpendicular lines, congruent and similar triangles, and circles. KY's College Readiness Standards expect only students whose intended majors require calculus and who expect to begin college taking calculus to be able to do geometric proofs—and those proofs are limited to key basic theorems and proofs related to congruent triangles, similar triangles, and circles. They set no expectations with respect to geometric constructions. Only minimal attention is given to proof in the K-12 standards documents, with the Core Content for Assessment making no mention of geometric proof and the Program of Studies making one reference. The Program of Studies and the Core Content for Assessment are, in fact, in conflict in the approach they take to congruent and similar triangles, with the Program of Study stating that students enrolled in high school geometry should be able to “prove triangles and other polygons congruent and similar” and the Core Content for Assessment stating that students are expected to be able to “apply the concepts of congruence and similarity to solve real-world and/or mathematical problems (not including proofs).” This variation needs to be reconciled. The Program of Studies for high school geometry references students being able to “integrate constructions,” naming a number of specific constructions, but it is not clear what is meant by the term “integrate.” The Core Content for Assessment makes no mention of constructions. This too needs to be reconciled.

- Geometric transformations are categorized as Level 3 expectations—essential only for students intending to enroll in college calculus in their freshmen year—in KY’s College Readiness Standards. Both the Program of Studies and the Core Content for Assessment place greater importance on transformations, with content statements at both the eighth-grade and high school levels addressing this content. The Core Content for Assessment indicates that transformations are eligible for assessment at grades 8 and 11.
- KY’s College Readiness Standards categorize expectations related to spatial visualization as Level 2—meaning that while this is valuable content, it can be acquired in college. KY’s Program of Studies and Core Content for Assessment both set expectations that call for spatial visualization, although it is less clear that the content statements in the Core Content for Assessment are consistent with what is intended by ADP and KY’s College Readiness Standards. Even though the College Readiness Standards place less priority on spatial visualization and reasoning than do the K-12 documents, it is important that Kentucky ensure that students entering directly into the workforce have the skills and knowledge they need. The state would be well-served to re-examine these expectations to ensure clarity and alignment.
- Neither of the KY K-12 documents sets the expectation that students be able to find the equation of a circle given its center and radius. This is an expectation of both the ADP benchmarks and the KY College Readiness Standards. While the ADP benchmarks also indicate that students should be able to find a circle’s center and radius given its equation, the KY College Readiness Standards define this as a skill needed only by students intending to take college calculus upon entry into college.
- The Program of Students for high school geometry states that students should explore concepts of vectors. It is not clear what “exploration” entails. Vectors are not addressed in any of the other documents.
- Many of the trigonometry benchmarks in ADP are asterisked, meaning they should be required for calculus-intending students. These same expectations are noted in KY’s College Readiness standards as Level 3 expectations—for students preparing to embark upon mathematics-intensive majors—which is consistent. Basic right triangle trigonometry is included in all of the documents, with the least explicit treatment being in the Program of Studies in which students are expected to “use right triangle relationships such as trigonometric ratios.” Basic trigonometric expectations—understanding and application of sine, cosine, and tangent to solve problems—are classified as Level 1 in KY’s College Readiness Standards.

## DATA ANALYSIS AND PROBABILITY

- While the ADP benchmarks and the KY College Readiness Standards are comparable in that they list the same expectations verbatim, they vary with respect to the student population they target. In the ADP benchmarks, all of the expectations are defined as essential for all students. In the KY College Readiness Standards, it is only the expectations that address the organization and

display of data, the reading and interpretation of basic data displays (tables, charts, and graphs), and the determination of basic summary statistics (mean, median, range, percentiles, variance, and standard deviation) that are noted as Level 1 expectations essential for all students if they expect to avoid placement in a remedial course. All of the other expectations are classified as Level 2 expectations meaning that they are of value but can be learned in college. While this may be the case, it is also the case—if the KY College Readiness Standards were to be used to define KY’s high school expectations—that those students who transition upon high school graduation not to college but to the world of work might never have the opportunity to learn data analysis and probability concepts that would be helpful to them in their personal and professional lives. Data analysis topics that are listed as Level 2 in the College Readiness Standards include: comparing data sets, understanding scatter plots, understanding the normal curve, evaluating and critiquing data published in the media, explaining misleading uses of data, differentiating between correlation and causation, using data to make conclusions or predictions, understanding sampling methods, identifying potential sources of bias, designing experiments or investigations, understanding the difference between randomized experiments and observational studies, and determining and interpreting a line of best fit. All expectations related to probability are listed as Level 2 expectations in the College Readiness Standards.

- Both the Program of Studies and the Core Content for Assessment list more extensive expectations for middle school and high school students with respect to data analysis and probability than the Level 1 KY College Readiness Standards. However, when focusing only on areas of commonality (i.e., data display and interpretation and summary statistics), there appears to be good alignment between the expectations set by the Program of Studies, the Core Content for Assessment, and the Level 1 expectations in the College Readiness Standards—even though it is difficult to be sure of this since the level of specificity varies considerably across the documents. For example, both the Program of Studies and the Core Content for Assessment detail the specific data displays students are to be able to construct and interpret, while the ADP benchmarks and KY College Readiness Standards are worded more generally.
- Some of the Level 2 College Readiness Standards have no comparable content statements in the Program of Studies and/or the Core Content for Assessment. For example, the K-12 documents make no mention of the normal distribution, understanding the difference between correlation and causation, or understanding the law of large numbers. This may signal agreement that these concepts are not essential for all students to know upon high school graduation—even though this contradicts the expectations set in the ADP benchmarks. On the other hand, a number of the Level 2 College Readiness Standards do have comparable expectations in one or both of the K-12 documents, pointing out an inconsistency in the value attributed to teaching probability and statistics concepts in middle and high school. Probability concepts, for example, are targeted in both middle and high school in the Program of Studies and the Core Content for Assessment, yet

probability is not defined as essential gateway content in the College Readiness Standards.

- The Core Content for Assessment offers a supporting content statement (not targeted for assessment) that high school students should be able to use matrices to represent real-world data and use matrix operations to solve problems. None of the other documents examined included expectations with respect to matrices.

## ALGEBRAIC THINKING

- There is strong alignment between the ADP benchmarks and KY's College Readiness Standards with respect to Algebra. Only one expectation—solving systems of three linear equations in three variables—is defined in the ADP benchmarks but not included in the College Readiness Standards. This expectation is noted with an asterisk in the ADP document to indicate that it is essential only for students planning mathematics-intensive majors. In another instance, an ADP benchmark is more extensive than its comparable College Readiness expectation—which states that students are to be able to graph the solution set of a linear inequality. The ADP benchmark extends this to include graphing the solution set of a system of two or three linear inequalities.
- The majority of ADP benchmarks identified as essential for students intending to take college calculus (denoted with asterisks) are defined in KY's College Readiness Standards as Level 2 or 3 expectations. Three such ADP benchmarks—addressing ellipses and hyperbolas, finite geometric series, and exponential functions whose solutions require facility with logarithms—are identified in the College Readiness Standards as Level 2 expectations that can be learned in college. Five such ADP benchmarks—addressing the domain of a function, combining functions, understanding functions and their inverses (including exponential functions and logarithms), and the binomial theorem—are identified as Level 3 College Readiness Standards essential to only those students intending mathematics-intensive majors. One asterisked ADP benchmark—dealing with the simplification of algebraic expressions containing rational exponents—was identified as an essential gateway (Level 1) expectation in the KY College Readiness Standards. One ADP benchmark identified as important for all students—recognizing and solving problems such as home mortgage problems that can be modeled using a finite geometric series—was classified in the KY College Readiness Standards as Level 2.
- Nine of the ten asterisked benchmarks in ADP that define content essential for only students intending to take college calculus are absent from KY's two K-12 documents. This signals agreement that these expectations are not important requirements for all high school students. One of the asterisked ADP benchmarks—the expectation that students be able to determine the domain of a function—is included in the Core Content for Assessment as an assessable content statement.
- Only one expectation defined as essential for all students in both the ADP benchmarks and KY's College Readiness Standards is missing from both KY K-12 documents. This is the expectation that students be able to evaluate

- polynomial and rational expressions and expressions containing radicals and absolute values. One expectation defined as essential for all students in the KY College Readiness Standards but as essential only for students intending to take college calculus in the ADP document is similarly missing from both K-12 documents. This is the expectation that students understand the properties of rational exponents and be able to apply them to simplify algebraic expressions.
- The Core Content for Assessment tends to be more rigorous and align more closely with both the ADP benchmarks and KY's College Readiness Standards than does the Program of Studies. The Program of Studies expectations with respect to algebra tend to place their heaviest emphasis on the linear aspects of algebra, so many expectations that are in the three other documents are left out. Topics in the Core Content for Assessment but omitted from the Program of Studies include the following: using the properties of integer exponents to simplify algebraic expressions; performing operations with polynomials, factoring polynomials by removing the greatest common factor, factoring quadratic polynomials, performing operations with and simplifying rational expressions, recognizing functions, determining the domain of a function, using function notation to evaluate a function, and solving systems of two linear equations.
  - Some of the content statements in both the Program of Studies and the Core Content for Assessment include so much content that it is unlikely that users of the documents will understand the full intent of the expectations. For example, the Program of Studies expects students to “use the skills learned to solve linear equations and inequalities to solve numerically, graphically, or symbolically non-linear equations such as quadratic and exponential equations.” The ADP benchmarks present this content in a way that is easier to digest, with separate benchmarks for solving, graphing, and modeling each of the various types of functions and equations. The language is clear, and it is easier to show—through examples attached to given benchmarks—what the level of expectation is.
  - While the Core Content for Assessment is the more rigorous of the two K-12 documents, it could benefit from further refinement. Some of its content statements include constraints which limit the rigor of the expectations relative to what is defined in the ADP benchmarks and the KY College Readiness Standards, and it would be a good idea to review these expectations. For example, the Core Content for Assessment document indicates that students should be able to factor quadratic polynomials of the form  $ax^2+bx+c$ , when  $a=1$  and  $b$  and  $c$  are integers. Both the ADP and the College Readiness Standards indicate that students should be able to factor quadratic polynomials—a more rigorous expectation since it does not place the same constraints on coefficients. In addition, it would also be a good idea to review the set of supporting content statements to ensure that none of them warrant assessment. For example, Kentucky might want to reconsider whether it is realistic and desirable to expect students to be able to write an explicit rule for the  $n$ th term of a geometric sequence, solve for a specified variable in a multivariable equation, or solve systems of linear inequalities on the eleventh-grade state assessment.